# Connectivity of Vehicular Networks Considering Underpass on a Highway

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***Abstract:*** **In this paper, in a highway scenario, the network connectivity problem is considered and developed a mathematical model to investigate the vehicles steady-state probability of connectivity in a one-directional road segment with one underpass. In the mathematical model, on the road the underpass is equally dispersed and distribute the road into two segments. The vehicles arrival follow Poisson process, on each segment. A vehicle drives towards the underpass with a specified probability. On the basis of these assumptions, we study various situations based on position distribution of the underpass, also takes into account the speed of vehicles, arrival rate of vehicles, impact of road length, and the probability of the vehicles driving towards the underpass and leave, in deriving the probability of connectivity. The derived mathematical model is validated over simulation results and the effects of numerous factors on the probability of connectivity are examined by both analytical and simulation results.**

***Keywords:*** **Vehicular ad hoc networks, underpass, probability of connectivity**

## I. INTRODUCTION

The most significant problem in vehicular ad hoc networks (VANETs) is connectivity of the network [1], [2]. Because of the dynamic movement of vehicles, the network connectivity often remains intermittent, which in result degrades the performance of network in terms of throughput and data delivery. This is the reason, that connectivity of network has been extensively considered for various road environments in VANET [3-15]. Most of the studies were considering the unrealistic simple highway scenarios without underpasses. Actually, in realistic scenarios, a highway has underpasses situated at various points. On each underpass, vehicles might arrive, depart, or stay on the road, which results in highly dynamic network connectivity. Hence, it is beneficial and interesting to investigate the connectivity of the network considering underpasses in a highway environment, and this motivates our work.

In this paper, in a highway scenario, the network connectivity problem is studied by considering underpass on a one-directional road segment. A mathematical model is developed to investigate the vehicles steady-state probability of connectivity on the road segment. In the proposed model, we suppose on the flyover underpass is equally dispersed and distribute the road into two segments. The arrival of vehicles follow Poisson process with various arrival rates on the two segments of road, respectively, and a vehicle drives towards the underpass with a specified probability. We study various situations based on position distribution of the underpass, also takes into account the speed of vehicles, arrival rate of vehicles, and the probability of the vehicles driving towards the underpass and leave, in deriving the probability of connectivity. The derived mathematical model is validated through simulation results and the effects of various factors on the probability of connectivity are individually examined by equally analytical and simulation results.

The rest of the paper is structured as follows.

Section 2 the related work is reviewed. Section 3 mathematical model is derived and the probability of connectivity is analyzed. Section 4 the accurateness of the mathematical model is confirmed over simulation results. Section 5 concludes the paper.

## II. LITERATURE REVEIW

In VANET for a highway scenario connectivity analysis has been extensively considered by the researchers [3-17]. The authors in [3], discussed that when the headway distance follows unrelated statistical distributions how the connectivity of the network in a highway environment will change. The authors in [4], proposed a mathematical model to study the vehicle's connectivity on a highway segment and revealed that when the network density is high even a trivial increase in communication range results in increasing connectivity probability. The authors in [5], studied the effects of vehicle communication range and vehicles average speed on the probability of connectivity in a one-dimensional network. The authors in [6], examined the interrelation between the probability of connectivity and the core parameters, such as network coverage of the regular vehicle and traffic density. Also highlighted the connectivity characteristics of the platoon based networks. The authors in [7], studied connectivity in the uni-directional highway road environment and presented an analytical model. To define the interrelation amongst the probability of connectivity, the model considered several parameters, such as traffic density, communication range, and vehicle speed distribution. According to the authors of [8], a mobile linear network was used as a platform for studying and evaluating the statistical properties of connectivity and the distribution of nodes in a steady state, under a strict delay constraint and increased mobility of nodes. However, the

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above-mentioned studies considered the performance of connectivity of an unremitting highway situation with no underpasses. The authors in [9], with multiple entries and exits, developed a performance model of connectivity in a highway scenario and given the nodes location distribution. Also, they focused on connectivity statistical properties, comprising a vehicle which is random that could see the entire vehicle population in a single cluster and the average clusters size. The authors in [10], proposed an methodical model to compute the two-hop downlink probability of connectivity, by considering the traffic distribution, road condition, and capability of vehicles. In our previous work [11], we highlighted the issues of network connectivity in a highway scenario considering toll plaza, where toll plaza could help the vehicles reduce the speed instead of blocking the flow of traffic to avoid traffic congestion. We developed a mathematical model to derive the probability of connectivity by taking into account various significant parameters, such as arrival rate of vehicles, speed, transmission range, and road length. Not the same as [11], in this paper, we considered the different use case scenario by taking into account underpasses on the highway.

## III. CONNECTIVITY MATHEMATICAL MODEL

In this section, we familiarize the system design and network scenario. Then we perform the connectivity probability analysis of the vehicles driving on the one-directional road segment considering underpass in detail.

### 3.1 System Design

As depicted in Figure 1, the system design considers a one-directional highway segment including one underpass equally dispersed on the road segment. We denote the road segment by $[\mathbf{0}, \boldsymbol{L}]$, where the road segment length is $\boldsymbol{L}$. The underpass is positioned at $\boldsymbol{y}(\boldsymbol{y}\,\boldsymbol{\epsilon}[\mathbf{0}, \boldsymbol{L}])$. The underpass can divide the whole segment of road into two sub-segments: $[\mathbf{0}, \boldsymbol{y}]$ and $[\boldsymbol{y}, \boldsymbol{L}]$. The arrival of vehicles follows a Poisson process on segment $[\mathbf{0}, \boldsymbol{y}]$ with a mean arrival rate $\boldsymbol{\lambda_1}$. Once a vehicle in $[\mathbf{0}, \boldsymbol{y}]$ drive towards the underpass, it will keep on driving with probability $\boldsymbol{\alpha}(\mathbf{0} \leq \boldsymbol{\alpha} \leq \mathbf{1})$ towards segment $[\boldsymbol{y}, \boldsymbol{L}]$ or with probability $\mathbf{1} - \boldsymbol{\alpha}$ leave the road. The vehicles availability on segment $[\boldsymbol{y}, \boldsymbol{L}]$ is based on two parts: one includes those vehicles which are traveling from segment $[\mathbf{0}, \boldsymbol{y}]$ and keep on traveling towards segment $[\boldsymbol{y}, \boldsymbol{L}]$; and the other one include those new vehicles which are coming from the underpass. The arrival of new vehicles follows a Poisson process with mean arrival rate $\boldsymbol{\lambda_2}$ on segment $[\boldsymbol{y}, \boldsymbol{L}]$. When the segment $[\boldsymbol{y}, \boldsymbol{L}]$ ends, all the vehicles will leave the road segment. Same as [8], suppose that at the location $\boldsymbol{y}(\mathbf{0} \leq \boldsymbol{y} \leq \boldsymbol{L})$ the speed of vehicles is a random variable on the road that is indicated as $\boldsymbol{v}(\boldsymbol{r})$. Hence, $\boldsymbol{v} = \boldsymbol{E}[\boldsymbol{v}(\boldsymbol{r})]$ indicates the average speed of the vehicles on the road, where the operation of mathematical expectation is given as $\boldsymbol{E}[\cdot]$.

By considering the assumptions above, based on [8], [9], it could be easy to achieve in a steady state that the arrival of vehicles on segment $[\mathbf{0}, \boldsymbol{y}]$ and segment $[\boldsymbol{y}, \boldsymbol{L}]$ with mean rates follow a Poisson process respectively. Also, we suppose that the vehicle transmission range is $\boldsymbol{V_r}$.

$$\gamma 1 \ \frac{\lambda_1}{v} \text{ and } \gamma 2 \ \frac{\alpha.\lambda_1+\lambda_2}{v}$$

### 3.2 problem statement

The problem of network connectivity is considered on the segment of road as depicted in Figure 1 and focusing on building a mathematical model to perform the network connectivity analysis of the vehicles traveling on the road segment. Here, two vehicles will be connected if the transmission range of a vehicle is greater than the distance amongst them. On the road segment, entire vehicles will be connected, if any two neighboring vehicles are connected. If all the vehicles on the road segment are connected, the segment will be connected. The network will be connected, when all the segments are connected.

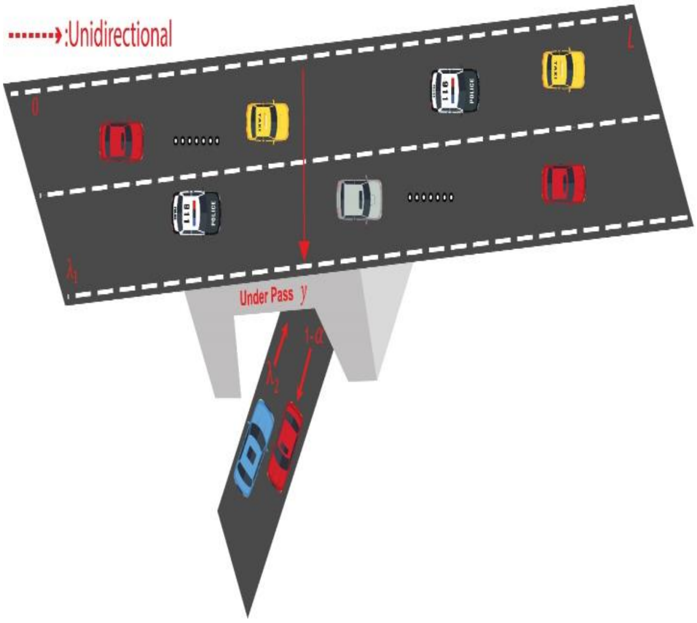


**Figure 1. System Model.**

In this section, we provide a detailed overview of the modelling of the COVID-19 cases dataset as a weighted two-mode (bipartite) network. Usually, the two-mode network comprises two disjoint sets of nodes namely primary (top) nodes $T$ and secondary (bottom) nodes. The nodes in the primary set establish the connection with the nodes of the secondary sets and vice versa. No pair of nodes have a connection in the same set of nodes [3]. However, there are many examples in the real world network such as actors-movie, authors-books, scientific collaboration networks, air transportation, etc. These networks are constructed using the graph theory. In the graph theory, the two-mode network is the triplet graph $G\ (T, \perp, \mathcal{L})$. Here, $\mathcal{L}$ is a set of links between top nodes $T$, and a set of bottom nodes $\perp$ [19].

To illustrate an unweighted two-mode network projection, two separate sets of nodes, namely the primary set of nodes 1, 2, 3, 4, 5, and 6, have a connection with nodes of the secondary set namely s, h, u, b, a, and n as shown in Fig. 2 (a). Here, node 1 has links with nodes S, and H, while node 2 has connections with nodes S, B, and N. Similarly, two nodes B and H (secondary set nodes) are linked with node 6 as depicted in Fig. 2 (a). To obtain the links between the nodes either primary nodes or secondary nodes, the two-mode network is projected onto the one-mode network by selecting either set of nodes. An example of the one-mode network projection by selecting a primary set of nodes is shown in Fig. 2 (b). The link is only established between the nodes if they have a common co-occurrence in the other set. Node 1 shares links with nodes 2, 5 and 6, due to co-occurrence with them in the other set. Node 5 is linked with nodes 1 and 4. Node 6 is linked with nodes 1, and 2, as illustrated in Fig. 2 (b). From the perspective of network analysis, it is essential to convert the network into one-mode network by selecting the desired set of nodes.

### 3.3 Connectivity Probability

According to Figure. 1, a one-directional highway is considered, since the underpass $\boldsymbol{y}$ is dispersed equally in $[\boldsymbol{0}, \boldsymbol{L}]$, hence, the connectivity probability will be analyzed under three situations:

Situation 1: the underpass $y$ is situated in $[0, V_r]$;
Situation 2: the underpass $y$ is situated in $[V_r, L - V_r]$;
Situation 3: the underpass $y$ is situated in $[L - V_r, L]$.

*Situation 1:* $y \in [0, V_r]$

In situation 1, the underpass $y$ is situated in $[0, V_r]$, i.e., $y \in [0, V_r]$. If the vehicles are driving on segment $[0, y]$, it is clear that entire vehicles are connected, because the transmission range $V_r$ is greater than the distance between any two vehicles. Now we have to consider whether the neighbor vehicles on both side of the underpass and the vehicles on the segment $[y, L]$ are connected. If the vehicles on segment $[0, y]$ are not available, then it is only important that the connectivity of segment $[y, L]$ should be analyzed.

The road $[0, L]$ is divided into three sub-segments for ease of analysis:
$[0, y]$, $[y, y + V_r]$ and $[y + V_r, L]$, which are denoted by $d_1, d_2,$ and $d_3,$ respectively as shown in Figure. 2.

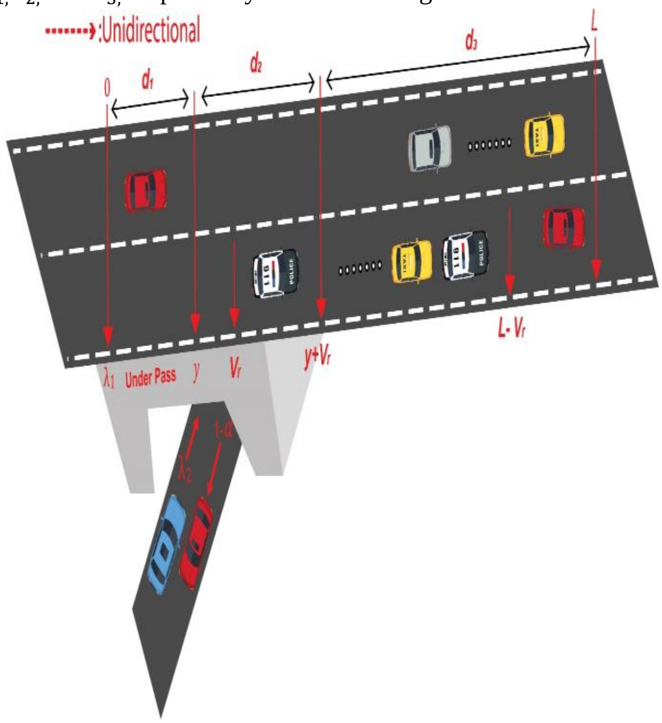


**Figure 2. The scenario of situation 1.**

The notations which are used in the analysis are given in Table 1.

**Table 1. Notations.**

| Symbol | Meaning |
|---|---|
| $H_x$: | the automobiles are roving on $x$ road segment. |
| $\overline{H_x}$: | the vehicles are not traveling on $x$ road segment. |
| $C_x$: | the $x$ road section is linked. |
| $\overline{C_x}$: | the $x$ road section is not connected. |
| $P_r\{X\}$: | the probability of happening event X. |

Such as, $H_{d2}$ indicate that vehicles are driving on segment $d_2$; i.e., segment $[y, y + V_r]$. $\overline{C_{d1}}$ indicates that the segment $d_1$, i.e., $[0, y]$, is not connected.

Since the arrival of vehicles follows the Poisson process on the road segment $[0, y]$, with parameter mean $\gamma 1$. Hence, the probability of having vehicles on segment $d_1$could be computed as:

$$\mathrm{P_r}\{H_{d1}\} = 1 - \frac{(\gamma 1 \,.\, y)^0}{0!} . e^{-\gamma 1 \,.\, y} = 1 - e^{-\gamma 1 \,.\, y} (1)$$

and the probability without any vehicle on $d_1$ is defined as

$$\mathrm{P_r}\{\overline{H_{d1}}\} = 1 - \mathrm{P_r}\{H_{d1}\} = e^{-\gamma 1 \,.\, y}. \quad (2)$$

also, the probabilities of the vehicles that are available on $d_2$ and $d_3$ are defined as

$$\mathrm{P_r}\{H_{d2}\} = 1 - e^{-\gamma 2 \,.V_r} \quad (3)$$

and

$$\mathrm{P_r}\{H_{d3}\} = 1 - e^{-\gamma 2 \,.(L-y-V_r)}, \quad (4)$$

respectively.

Therefore, the probabilities of no vehicles on $d_2$ and $d_3$ are defined as

$$\mathrm{P_r}\{\overline{H_{d2}}\} = e^{-\gamma 2 . V_r} \quad (5)$$

and

$$\mathrm{P_r}\{\overline{H_{d3}}\} = e^{-\gamma 2 .(L-y-V_r\,)} \quad (6)$$

respectively.

We perform the analysis by dividing situation 1 into further two sub-situations:

Situation 1.1: on segment $d_1$ vehicles are driving.
Situation 1.2: on segment $d_1$ vehicles are not driving.

*Situation 1.1.*

In situation 1.1, the vehicles driving on the segment $d_1$ are connected. Then the analysis of the connectivity probability will be conducted based on the conditions that on segment $d_2$ vehicles are driving and no vehicles are driving on $d_2$, respectively.

On segment $d_2$ vehicles are driving.

If on segment $d_2$ vehicles are driving, they will be considered connected on segment $d_2$. According to [9], if two vehicles are connected with each other, which are located on segment $d_1$ and $d_2$, then $d_1$ and $d_2$ are connected. Therefore, we conduct the probability analysis of two vehicles which are located at $d_1, d_2$ and are connected with each other respectively.

Hence, the probability of two segments $d_1$ and $d_2$ that are interconnected with each other could be computed as [9]

$$\Pr\{C_{[0,y+V_r]} | H_{d1}, H_{d2}\} = 1 - e^{-\gamma 2.V_r.(1-q1)} \quad (7)$$

where

$$q1 = \frac{e^{0.5.\gamma 1.y^2/V_r} - 1}{e^{\gamma 1.y} - 1} \quad (8)$$

Further, in the situation when vehicles driving on segment $d_2$ and the probability that segment $[y, L]$ is connected could be computed as:

$$\begin{aligned}\Pr\{C_{[y,L]} | H_{d2}\} &= \Pr\{C_{[y,L]}, H_{d2}\} / \Pr\{H_{d2}\} \\ &= \Pr\{C_{[y,L]}\}. \Pr\{H_{d2} | C_{[y,L]}\} / \Pr\{H_{d2}\} \\ &= \Pr\{C_{[y,L]}\}. (1 - \Pr\{\overline{H_{d2}} | C_{[y,L]}\}) / \Pr\{H_{d2}\}, \\ &= \Pr\{C_{[y,L]}\}. (1 - \Pr\{C_{d3}\}) / \Pr\{H_{d2}\}\end{aligned} \quad (9)$$

In the above equations $\Pr\{C_{[y,L]}\}$ represents the probability of the vehicles which are connected on the road segment $[y, L]$ and $\Pr\{C_{d3}\}$ represents the probability of vehicles which are connected on segment $d_3$. According to [13], for a highway, if arrival of vehicles follow a Poisson process, then the connectivity probability of the road segment is defined as

$$\Pr\{\lambda, V_r, L\} = e^{-\lambda L} \sum_{j=0}^{\lfloor L/V_r \rfloor} \frac{(-1)^j}{j!} [\lambda(L - jV_r)]^{j-1} \times [j + \lambda(L - jV_r)] e^{\lambda.(L - jV_r)}, \quad (10)$$

where the road segment length is $L$. Hence, we get

$$\Pr\{C_{[y,L]}\} = P(\gamma_2, V_r, L - y), \quad (11)$$

$$\Pr\{C_{d3}\} = P(\gamma_2, V_r, L - y - V_r). \quad (12)$$

Hence, the probability that the segment of road $[0, L]$ is connected when vehicles are driving on $d_2$ is defined as

$$p_{111} = \Pr\{C_{[0,y+V_r]} | H_{d1}, H_{d2}\}. \Pr\{C_{[y,L]} | H_{d2}\}. \quad (13)$$

On segment $d_2$ vehicles are not driving.

If on segment $d_2$ vehicles are not driving, the road segment $[0, y + V_r]$ will be not connected obviously. In that situation, the road $[0, L]$ is connected occur only while vehicles are not driving on $d_3$. Therefore, in the absence of vehicles on $d_2$ the probability of the road $[0, L]$ is connected is defined as

$$p_{112} = \Pr\{\overline{H_{d3}}\}. \quad (14)$$

Thus, we could get the probability of the section $[0, L]$ is connected in situation 1.1, i.e.,

$$p_{11} = \Pr\{H_{d2}\}. p_{111} + \Pr\{\overline{H_{d2}}\}. p_{112}. \quad (15)$$

*Situation 1.2.*

In situation 1.2, the vehicles are not driving on the segment $d_1$, in this situation, we have to consider connectivity on segment $[y, L]$ only. To achieve this, the analysis of the connectivity probability will be conducted based on the conditions that on segment $d_2$ vehicles are driving and no vehicles are driving on $d_2$, respectively.

$$p_{121} = \Pr\{C_{[y,L]} | H_{d2}\}, \quad (16)$$

Where $\Pr\{C_{[y,L]} | H_{d2}\}$ is defined by (9).

In the situation, when there are no vehicles driving on segment $d_2$, the segment of road $[0, L]$ is connected occurs is equals to road segment $d_3$ is connected. Hence, the road's $[0, L]$ connectivity probability when there are no vehicles driving on $d_2$ is defined as

$$p_{122} = \Pr\{C_{d3}\} = \Pr\{\gamma_2, V_r, L - y - V_r\}, \quad (17)$$

and in situation 1.2 the probability of segment $[0, L]$ is connected could be defined as

$$p_{12} = \Pr\{H_{d2}\}. p_{121} + \Pr\{\overline{H_{d2}}\}. p_{122}. \quad (18)$$

Based on all the situations above, the connectivity probability in situation 1 could be defined as

$$p_1 = \Pr\{H_{d2}\}. p_{11} + \Pr\{\overline{H_{d1}}\}. p_{12}. \quad (19)$$

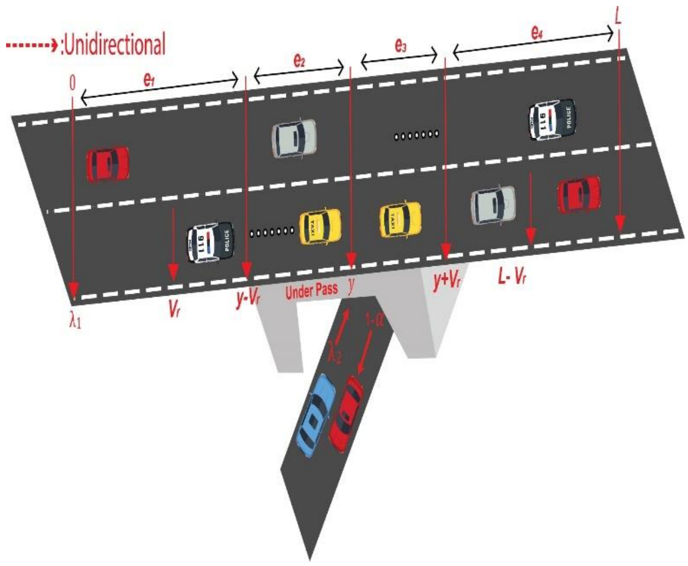


**Figure 3. Scenario of situation 2.**

*Situation 2: $y \in [V_r, L - V_r]$*

Same as situation 1, the road $[0, L]$ is divided into further four sub-segments: $[0, y - V_r]$ $[y, -V_r, y]$, $[y, y + V_r]$ and $[y + V_r, L]$, which are denoted by $e_1$, $e_2$, $e_3$, and $e_4$, respectively as shown in Figure. 3. Hence, the below given probabilities could be obtained easily:

$$\Pr\{H_{e1}\} = 1 - e^{-\gamma 1.(y - V_r)}, \quad (20)$$
$$\Pr\{H_{e2}\} = 1 - e^{-\gamma 1.V_r}, \quad (21)$$
$$\Pr\{H_{e3}\} = 1 - e^{-\gamma 2.V_r}, \quad (22)$$
$$\Pr\{H_{e4}\} = 1 - e^{-\gamma 2.(L - y - V_r)}, \quad (23)$$
$$\Pr\{\overline{H_{e1}}\} = e^{-\gamma 1.(y - V_r)}, \quad (24)$$
$$\Pr\{\overline{H_{e2}}\} = e^{-\gamma 1.V_r}, \quad (25)$$
$$\Pr\{\overline{H_{e3}}\} = e^{-\gamma 2.V_r}, \quad (26)$$
$$\Pr\{\overline{H_{e4}}\} = e^{-\gamma 2.(L - y - V_r)}. \quad (27)$$

on segment $e_2$, if vehicles are driving, then transmission range $V_r$ between any two vehicles is greater than the distance amongst any two vehicles. Hence, on segment $e_2$, all the vehicles are connected. Segment $e_3$ carrying similar properties as $e_2$. In this situation, the connectivity of $e_2$ and $e_3$ plays an essential part in the connectivity of the road $[0, L]$. By considering the observations above, situation 2 is divided into below given four sub-cases and the analysis is performed respectively:

Situation 2.1: vehicles are not driving on both segments $e_2$ and $e_3$;

Situation 2.2: vehicles are driving on $e_2$, but vehicles are not driving on $e_3$ ;

Situation 2.3: vehicles are not driving on $e_2$, but vehicles are driving on $e_3$ ;

Situation 2.4: on both segments, $e_2$ and $e_3$ vehicles are driving.

*Situation 2.1.*

In situation 2.1, the vehicles are not driving on both of the segments $e_2$ and $e_3$, in this situation, both of the segments $e_2$ and $e_3$ are not connected. If the vehicles are not driving on $e_4$ and $e_1$ or if the vehicles are not driving on $e_1$ and $e_4$ is connected. Therefore, the road's $[0, L]$ connectivity probability in situation 2.1 is defined as

$$p_{21} = \Pr\{C_{e1}\} . P_r\{\overline{H_{e4}}\} + \Pr\{C_{e4}\}. \Pr\{\overline{H_{e1}}\}, \qquad (28)$$

Where

$$\Pr\{C_{e1}\} = P\{ \lambda_1, V_r, y - V_r\}, \qquad (29)$$

$$\Pr\{C_{e4}\} = P\{ \lambda_2, V_r, L - y - V_r\}. \qquad (30)$$

*Situation 2.2.*

In situation 2.2, the vehicles are driving on the segment $e_2$, but vehicles are not driving on $e_3$. In this situation, $e_2$ is connected but $e_3$ is not connected. In the meantime, both of the segments $e_2$ and $e_3$ are not connected with each other. When vehicles are not driving on $e_4$ and segment $[0, y]$ is connected based on the condition that vehicles are driving on $e_2$, the segment of road $[0, L]$ is connected. Hence, the probability of road $[0, L]$ is connected in situation 2.2 could be defined as

$$p_{22} = \Pr\{C_{[0,y]}|H_{e2}\} . P_r\{\overline{H_{e4}}\}, \qquad (31)$$

where

$$\Pr\{C_{[0,y]}|H_{e2}\} = \Pr\{C_{[0,y]}\} . (1 - \Pr\{C_{e1}\})/\Pr\{H_{e2}\}$$

$$P( \quad \gamma 1, V_r, y). [1 - p(\gamma 1, V_r, y - V_r)]/\Pr\{H_{e2} \qquad (32)$$

*Situation 2.3.*

Same as situation 2.2, the probability of road $[0, L]$ is connected in situation 2.3 could be defined as

$$p_{23} = \Pr\{C_{[y,L]}|H_{e3}\} . P_r\{\overline{H_{e1}}\}, \qquad (33)$$

where

$$\Pr\{C_{[y,L]}|H_{e3}\} = \Pr\{C_{[y,L]}\} . (1 - \Pr\{C_{e4}\})/\Pr\{H_{e3}\}$$
$$= P( \quad \gamma 2, V_r, L - y). [1 - p(\gamma 2, V_r, L - y - V_r)]/\Pr\{H_{e3}\} \qquad (34)$$

*Situation 2.4.*

In situation 2.4, the vehicles are driving on both of the segments $e_2$ and $e_3$, in this situation, the probability analysis of $e_2$ and $e_3$ is conducted, which shows they are connected to each other. According to [9], it is defined as

$$\Pr\{C_{[y-V_r,y+V_r]}|H_{e2}, H_{e3} \} = 1 - e^{-\gamma 2 .vr.(1-q2)} \qquad (35)$$

where

$$q2 = \frac{e^{0.5.\gamma 1.vr} - 1}{e^{\gamma 1.vr} - 1}. \qquad (36)$$

Further, the probability of the segment $[0, y]$ is connected based on the situation that vehicles are driving on $e_2$ could be computed by Eq. (32). The probability of the segment $[y, L]$ is connected based on the situation that vehicles are driving on $e_3$ could be computed by Eq. (34). Hence, the probability of road $[0, L]$ is connected in situation 2.4 could be defined as

$$p_{24} = \Pr\{C_{[y-V_r,y+V_r]}|H_{e2}, H_{e3} \}. \qquad (37)$$

Based on all the situations above, the probability of connectivity in situation 2 could be computed as

$$p_2 = \Pr\{\overline{H_{e2}}\}. \Pr\{\overline{H_{e3}}\}. p_{21} + \Pr\{H_{e2}\}. \Pr\{\overline{H_{e3}}\}. p_{22} + \Pr\{\overline{H_{e2}}\}. \Pr\{H_{e3}\}. p_{23} + \Pr\{H_{e2}\}. \Pr\{H_{e3}\}. p_{24} \qquad (38)$$

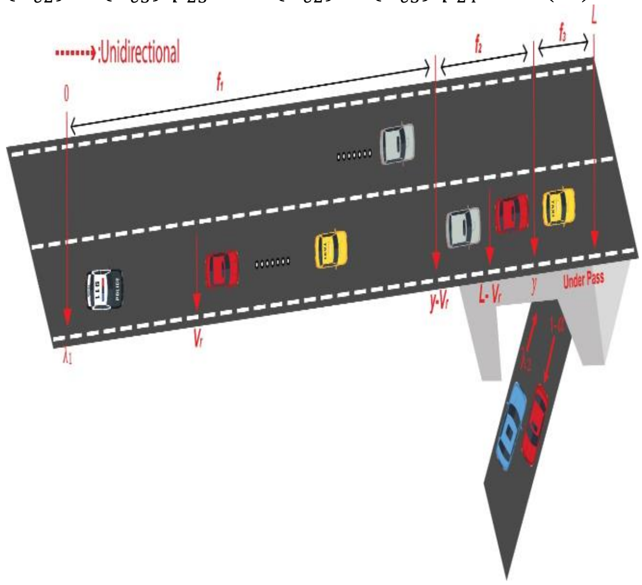


**Figure 4. The scenario of situation 3.**

*Situation 3: $y \in [L - V_r, L]$*

Same as situation 1, the road $[0, L]$ is divided into three sub-segments: $[0, y - V_r]$ $[y, -V_r, y]$, and $[y, L]$, which are denoted by $f_1$, $f_2$, and $f_3$ respectively as shown in Figure. 4. Hence, the below given probabilities could be obtained easily:

$$\Pr\{H_{f1}\} = 1 - e^{-\gamma 1 .(y-V_r)}, \qquad (39)$$
$$\Pr\{H_{f2}\} = 1 - e^{-\gamma 1 .V_r}, \qquad (40)$$
$$\Pr\{H_{f3}\} = 1 - e^{-\gamma 2 .(L-y)}, \qquad (41)$$
$$\Pr\{\overline{H_{f1}}\} = e^{-\gamma 1 .(y-V_r)}, \qquad (42)$$
$$\Pr\{\overline{H_{f2}}\} = e^{-\gamma 1 .V_r}, \qquad (43)$$
$$\Pr\{\overline{H_{f3}}\} = e^{-\gamma 2 .(L-y)}. \qquad (44)$$

We perform further analysis, by dividing situation 3 into two sub-situations:

Situation 3.1: on segment $f_3$ vehicles are driving;

Situation 3.2: on segment $f_3$ vehicles are not driving.

*Situation 3.1.*

In situation 3.1, the vehicles are driving on the segment $f_3$ are connected. Then the analysis of the connectivity probability will be conducted based on the conditions that on segment $f_2$ vehicles are driving and no vehicles are driving on $f_2$, respectively.

On segment $f_2$ vehicles are driving.

If on segment $f_2$ vehicles are driving, they will be considered connected on segment $f_2$. In this situation, if segment $f_2$ and $f_3$ are connected with each other, the segment of road$[y - V_r, L]$ is connected. Hence, the probability of road segment $[y - V_r, L]$ is connected could be computed as [9]

$$\Pr\{C_{[y-V_r,L]} | H_{f2}, H_{f3}\} = 1 - e^{-\gamma 1.V_r.(1-q3)}, \quad (45)$$

where

$$q3 = \frac{e^{0.5.\gamma 2.(L-y)^2/vr}{}_{-1}}{e^{\gamma 2.(L-y)}{}_{-1}}. \quad (46)$$

Further, the probability of the segment $[0, y]$ is connected based on the situation that vehicles are driving on $f_2$ could be computed as

$$\Pr\{C_{[0,y]} | H_{f2}\} = P(\gamma 1, V_r, y).[1 - p(\gamma 1, V_r, y - V_r)]/\Pr\{H_{f2}\}. \quad (47)$$

Hence, the road's $[0, L]$ connectivity probability when there are vehicles driving on $f_2$ is defined as

$$p_{311} = \Pr\{C_{[y-V_r,L]} | H_{f2}, H_{f3}\}.\Pr\{C_{[0,y]} | H_{f2}\}. \quad (48)$$

On segment $f_2$ vehicles are not driving.

If on segment $f_2$ vehicles are not driving, the road segment $[y - V_r, L]$ will not be connected obviously. In that situation, the road $[0, L]$ is connected occur only while vehicles are not driving on $f_1$. Therefore, in the absence of vehicles on $f_2$ the probability of the road $[0, L]$ is connected is defined as

$$p_{312} = \Pr\{\overline{H_{f1}}\}. \quad (49)$$

Thus, we could get the probability of the segment $[0, L]$ is connected in situation 3.1, i.e.,

$$p_{31} = \Pr\{H_{f2}\}.p_{311} + \Pr\{\overline{H_{f2}}\}.p_{312}. \quad (50)$$

*Situation 3.2.*

In situation 3.2, the vehicles are not driving on the segment $f_3$, in this situation, we have to consider connectivity on segment $[0, y]$ only. To achieve this, the analysis of the connectivity probability will be conducted based on the conditions that on segment $f_2$ vehicles are driving and no vehicles are driving on $f_2$, respectively.

Hence, the road's $[0, L]$ connectivity probability when there are vehicles driving on $f_2$, is defined as

$$p_{321} = \Pr\{C_{[0,y]} | H_{f2}\}, \quad (51)$$

where

$\Pr\{C_{[0,y]} | H_{f2}\}$ is defined by Eq. (47).

In the situation, when there are no vehicles driving on segment $f_2$, the segment of road $[0, L]$ is connected occurs is equals to road segment $f_1$ is connected. Hence, the road's $[0, L]$ connectivity probability when there are no vehicles driving on $f_2$ is defined as

$$p_{322} = \Pr\{C_{f1}\} = P(\gamma 1, V_r, L - V_r), \quad (52)$$

and in situation 3.2 the probability of segment $[0, L]$ is connected could be computed as

$$p_{32} = \Pr\{H_{f2}\}.p_{321} + \Pr\{\overline{H_{f2}}\}.p_{322.} \quad (53)$$

Based on all the situations above, the probability of connectivity in situation 3 could be computed as

$$p_3 = \Pr\{H_{f3}\}.p_{31} + \Pr\{\overline{H_{f3}}\}.p_{32}. \quad (54)$$

Hence, the probability of the road segment $[0, L]$ is connected based on the situation that there is one underpass equally dispersed in $[0, L]$ could be computed as

$$p = \frac{1}{L}.\left(\int_0^{V_r} p_1 L_y + \int_{V_r}^{L-V_r} p_2 L_y + \int_{L-V_r}^{L} p_3 L_y\right). \quad (55)$$

## IV. EXPERIMENTAL RESULTS AND ANALYSIS

In this section, the effectiveness and exactness of the proposed mathematical model is proven by simulations. In addition, the influence of different parameters on the performance of connectivity is discussed and analyzed, including the vehicles arrival rate $\lambda_1$ and $\lambda_2$, the speed $v$ and the probability $\alpha$ that the vehicles driving on road $[0, y]$ keep on driving towards road segment $[y, L]$. The analytical results were achieved by the derived mathematical model using MATLAB. We assumed in the simulations that the transmission range of every vehicle is $V_r = 300$ m, and the road length is 10 Km. All the simulations are based on $10^5$ average trials.

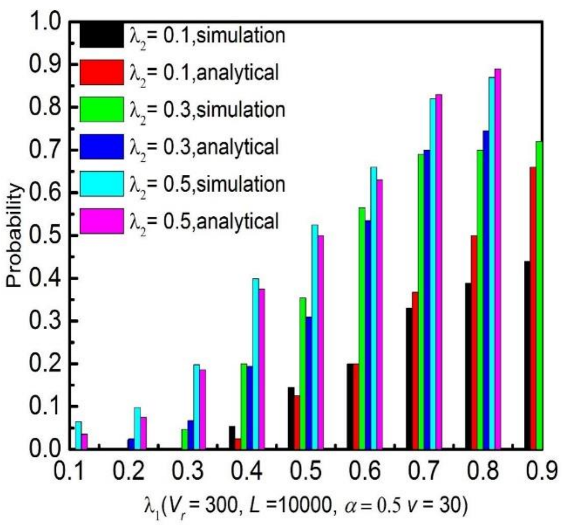


**Figure.5. Connectivity probability in contrast with $\lambda_1$ and $\lambda_2$.**

The impacts of $\lambda_1$ and $\lambda_2$ in the considered situation is shown in Figure. 5. In which we set $v = 31$ m/s, $V_r = 300$ m, $\alpha = 0.5$, and $L = 10000$ m. It could be seen that results obtained by simulations are close to the mathematical results, which shows the mathematical model is correct. It is observed in the given situation, with the increase in $\lambda_1$ and $\lambda_2$, the road's connectivity probability increases. Which shows that a high arrival rate could increase the entire network connectivity probability.

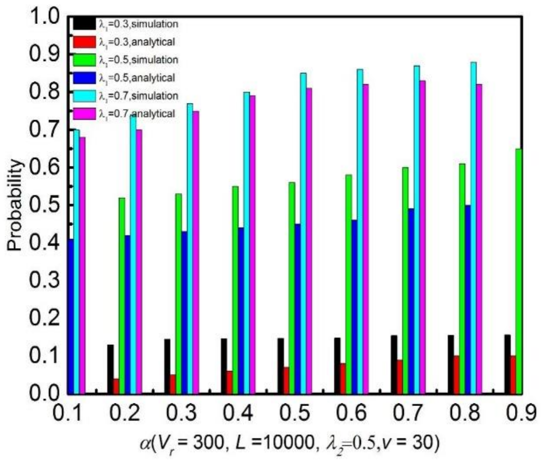


**Figure.6. Connectivity probability in contrast with $\lambda_1$ and $a$.**

The impacts of $\lambda_1$ and $a$ in the considered situation is shown in Figure. 6. In which we set $v = 31$ m/s, $V_r = 300$ m, $\lambda_2 = 0.5$ veh/s, and $L = 10000$ m. It is observed that in our situation with the increase in $a$ the road's connectivity probability increases. The reason is that, with the increase in $a$, many vehicles drive towards the underpass and leave, which in result increases $\gamma_2$ also increases the road's connectivity probability.

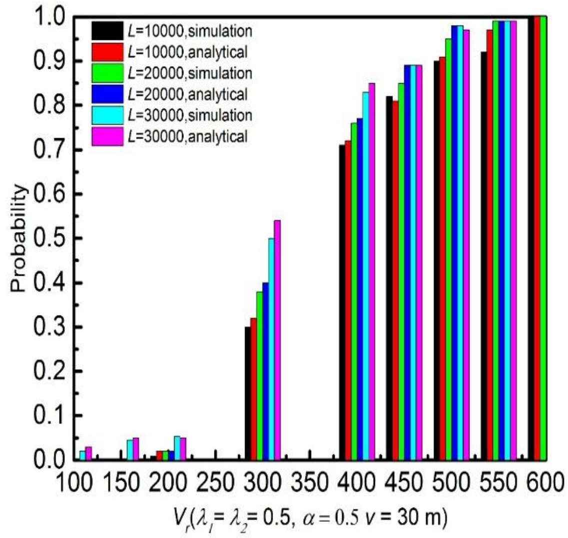


**Figure.7. Connectivity probability in contrast with $V_r$ and $L$.**

The impacts of $V_r$ and $L$ in the considered situation, is shown in Figure. 7. In which we set $v = 31$ m/s, $a = 0.5$ and $\lambda_1 = \lambda_2 = 0.5$ veh/s. It is observed that in both of the scenarios with the increase in $L$ the roads connectivity probability decreases. The reason is that, when the arrival rate of vehicles is not changing, a large value of $L$ causes few vehicles in a unit length of the road also a large distance between two consecutive vehicles. Which results in a decrease of connectivity probability.

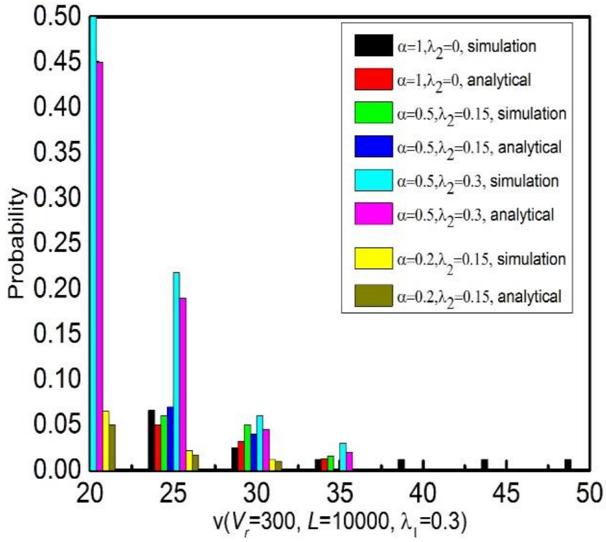


**Figure. 8. Connectivity probability in contrast with $v, a$ and $\lambda_2$.**

The impacts of $v, a$ and $\lambda_2$ in the considered situation, is shown in Figure. 8. In which we set $V_r = 300$ m, $L = 10000$ m, and $\lambda_1 = 0.3$ veh/s.

In the first situation where $\lambda_2 = 0$, $a = 1$, that shows there are no vehicles entering or leaving the underpass. That equals to the situation that no underpass, exists on the road. In this situation, the arrival rate of the vehicles remains 0.3 veh/s on the road. In the second situation where $\lambda_2 = 0.15$ and $a = 0.5$, that shows with probability 0.5 on segment $[0, L]$ the vehicles drive towards the underpass. In this situation the arrival rate of vehicles on segment $[y, L]$ remains 0.3 veh/s that is equivalent to $\lambda_1$. Therefore, in the two situations vehicles rate of arrival on the road is equal. As shown in Figure. 8, the roads connectivity probability as compared to the second situation, is higher in the first situation. It is now clear that the presence of the underpass has an influence on the road's connectivity probability. In the third situation, where $\lambda_2 = 0.3$ and $a = 0.5$, it is clear that the connectivity probability as compare to second situation is higher. The reason is that in this situation $a$ is equivalent to that in the second situation but $\lambda_2$ is higher. As a result, the arrival rate of vehicles on segment $[y, L]$ is higher than the second situation, which results in a higher connectivity probability. In the fourth situation, where $\lambda_2 = 0.15$, and $a = 0.2$, as compared to the second situation the connectivity probability is smaller. The reason is that in this situation $a$ is smaller to that in the second situation but $\lambda_2$ is equivalent. As a result, the vehicles arrival rate on segment $[y, L]$ is lesser as compared to the second situation, which results in a lesser probability of connectivity.

## V. CONCLUSION

In this paper, with one underpass, the steady-state connectivity probability of a one-directional highway road scenario is considered. To analyze the vehicles connectivity probability on the road a mathematical model was developed, in which various situations according to the location distribution of the underpass are considered. The various parameters are considered that could influence the road's connectivity probability, vehicle's arrival rate, traveling speed, road length, and the probability that the vehicle's drive towards the underpass. The simulation results confirm the efficiency of the mathematical model also ensures that the method could be used to analyze the influences of different parameters on the probability of connectivity. The more practical scenario will be considered as future work.

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