Linear Stability Analysis of Two Dimensional MHD Unsteady Flow of Viscous Fluid on a Shrinking Sheet

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Abstract— In this article, the work of Lund et al. (2019) has been extended for stability analysis, which was not considered in their study. In this study, the stability analysis of dual solutions for Caputo fractional-order-two dimensional MHD generalized viscous fluid over a shrinking sheet has been considered. The system of governing partial differential equations is reduced to the linearized system of eigenvalue problems. The resultant equations have been solved by using three stages Lobatto IIIa formula. The results revealed that the first solution is more stable as compared to the second solution, as expected. Further, it has been observed that the behavior of the initial growth of the disturbance is noticed for the unstable solution.

Keywords— Stability Analysis; Dual solutions; three stages Lobatto III a formula; Stable solution.

I. INTRODUCTION

Presently a day, nonlinear restrict esteem issues (BVPs) of liquid streams, and their more than a few arrangements are a crucial piece of topics in designing, material science, and arithmetic. To be sure, due to the fact of their vast software in designing and logical research. Dero, Rohni, and Saaban (2019) found the numerous arrangements of steady MHD movement of micropolar nanofluid over the exponentially extending and contracting surface and saw that triple preparations exist when a floor is contracting with high estimations of suction and the fascinating parameters. Lund et al., (2019a) viewed the micropolar nanofluid and observed the triple arrangement by using utilizing the BVP4C approach in MATLAB and determined that the necessary arrangement is steady. Raju et al., (2016) referenced that double arrangement exists just in a unique scope of Power-law list esteems and observed that limit in the warmness cross and coefficient of skin grating because of augmentations in an area of a pleasing parameter. Weidman and Ishak (2015) found numerous arrangements in two-and three-dimensional progressions of liquid and introduced that a regular arrangement is the top branch arrangement. Different arrangements exist in liquid circulation troubles due to non-linearity in the overseeing prerequisites (Lund, Omar, and Khan, 2019). Besides, numerous analysts forget about to see a range of arrangements because of their shakiness behavior (Raza et al., 2016). From the evaluation of special arrangements, it very well may also be reasoned that the solidness examination of numerous arrangements is essential to understand the consistent arrangement.

As it is referenced, numerous arrangements are considered in the liquid movement trouble because of their large scope of uses in various zones of businesses, designing, etc. To understand which arrangement is steady and possible and which one is not steady and now not viable truly. For this, we have to think about the dependability examination. It seems that Merkin (1986) was first who determined a double arrangement and did steadiness in the circulation trouble of blended convection in a porous medium and carried out soundness to show the foremost arrangement is continuously steady. From that point forward, Rosca and Pop (2014) proceeded with it in their examination and observed that solitary the top association is constant and feasible. Lund, Zurni, and Khan (2019a) performed out the solidness examination for the Casson liquid and seen that just a single association is steady. The same creators regarded the Maxwell liquid over the exponential penetrable surface and observed that simply a single association is uniform shape double preparations (Lund, Zurni and Khan, 2019b). Rana, Shukla, Gupta, and Pop (2019) noticed quite a several preparations by way of utilizing of homotopy investigation approach and located that lone first association can be seen tentatively. As it tends to be seen that one-of-a-kind preparations can not be imagined through examination from the considerable writing audit. In such a manner, the numerical investigation of solidness examination is vital and ought to be regarded in the investigations of special arrangements. Consequently, the top goal of the present investigation is to play out the soundness examination of crafted by Lund et al., (2019). Typically, these outcomes would contribute and provide productive consequences in the advancement of science and innovation.

II. DERIVATION OF STABILITY ANALYSIS

At the point when extra than one association exists in any liquid flow trouble, then it is vital to function protection examination. As per Lund et al., (2019b) and Weidman,

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Kubitschek, and Davis (2006), the given arrangement of conditions ought to be changed over to flimsy structure with the aid of imparting another time-subordinate dimensionless parameter. In this regard, we have τ=ct/(1-γt) for the current problem. Further, we have the following governing equations in the form of unsteady flow.

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\vartheta\frac{\partial^2 u}{\partial y^2}-\frac{\sigma^* B^2 u}{\rho} \quad (1)$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2 \quad (2)$$

where respective velocities of x and y-axis are u and v, ϑ is the kinematic viscosity, σ* is the Stefan-Boltzmann constant,ρ is the density of fluid, the transverse magnetic field of strength B=B_0/(1-γt)^(1⁄2) is applied with the normal to surface direction, T is the temperature, α is the thermal diffusivity, and μ is the dynamic viscosity.

With the new dimensionless similarity variable τ, we have

$$u=\frac{cx}{(1-\gamma t)}\frac{\partial f(\eta,\tau)}{\partial \eta};v=-\sqrt{\frac{c\vartheta}{(1-\gamma t)}}f(\eta,\tau);\eta=\sqrt{\frac{c}{\vartheta(1-\gamma t)}}y;\theta(\eta,\tau)=\frac{T-T_\infty}{T_w-T_\infty}\tau=\frac{ct}{(1-\gamma t)} \quad (3)$$

By substituting equation (3) in equations (1-2), we get

$$\frac{\partial^3 f}{\partial \eta^3}-\left(\frac{\partial f}{\partial \eta}\right)^2+f\frac{\partial^2 f}{\partial \eta^2}-A\left(\frac{\eta}{2}\frac{\partial^2 f}{\partial \eta^2}+\frac{\partial f}{\partial \eta}\right)-M\frac{\partial f}{\partial \eta}-(1+A\tau)\frac{\partial^2 f}{\partial \tau \partial \eta}=0 \quad (4)$$

$$\frac{1}{Pr}\frac{\partial^2 \theta}{\partial \eta^2}+f\frac{\partial \theta}{\partial \eta}-m\theta\frac{\partial f}{\partial \eta}-A\left(\frac{\eta}{2}\frac{\partial \theta}{\partial \eta}+\theta\right)+Ec\left(\frac{\partial^2 f}{\partial \eta^2}\right)^2-(1+A\tau)\frac{\partial \theta}{\partial \tau}=0 \quad (5)$$

Subject to boundary conditions

$$\begin{cases} f(0,\tau)=S,\dfrac{\partial f(0,\tau)}{\partial \eta}=-1,\theta(0,\tau)=1 \\ \dfrac{\partial f(\eta,\tau)}{\partial \eta}=\theta(\eta,\tau)=0 \quad as \quad \eta\to\infty \end{cases} \quad (6)$$

here A=γ/c is the unsteadiness parameter (Dero, Uddin, and Rohni, 2019). In our case, a decelerating shrinking surface (A<0). Further, $Ec=\frac{U_w^2}{C_p(T_w-T_\infty)}$, $Pr=\frac{\vartheta}{\alpha}$, and$M=\frac{\sigma(B_0)^2}{\rho c}$

are Eckert, Prandtl numbers,and magnetic parameter.

To check the stability of unsteady flow solutions where $f(\eta)=f_0(\eta)$ and $\theta(\eta)=\theta_0(\eta)$ of satisfying the boundary value problem of Lund et al., (2019), we have

$$\begin{cases} f(\eta,\tau)=f_0(\eta)+e^{-\gamma\tau}F(\eta,\tau) \\ \theta(\eta,\tau)=\theta_0(\eta)+e^{-\gamma\tau}G(\eta,\tau) \end{cases} \quad (7)$$

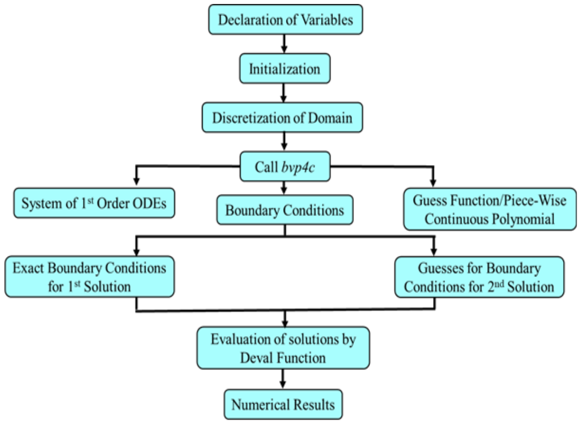

Chart 1: Flow chart of numerical method.

where *F(η)*, and *G(η)* are small relative of $f_0(\eta)$, and $\theta_0(\eta)$ respectively. Further, γ is the unknown eigen values. Substituting the relations (7) in (4-5), the following equations are obtained:

$$F_0'''-2f_0'F_0'+f_0F_0''+F_0f_0''-A\left(\frac{\eta}{2}F_0''+F_0'\right)-MF_0'+\gamma F_0'=0 \quad (8)$$

$$\frac{1}{Pr}G_0''+f_0G_0'+F_0\theta_0'-m(G_0f_0'+\theta_0F_0')--A\left(\frac{\eta}{2}G_0'+G_0\right)+\gamma G_0=0 \quad (9)$$

subject to boundary condition

$$\begin{cases} F_0(0)=0,F_0'(0)=0,G_0(0)=0 \\ F_0'(\eta)\to 0,\ \ G_0(\eta)\to 0 as\ \eta\to\infty \end{cases} \quad (10)$$

It is worth mentioning that there exist an infinite set of eigenvalues $\gamma_1<\gamma_2<\gamma_3$....where γ_1 is the smallest eigenvalue. According to Lund, Omar, Khan, and Dero (2019), in order to perform the stability of (8-9) we need to relax one boundary condition on $F'_0(\eta)$ and $G_0(\eta)$. In current problem,$F'_0(\eta)\to 0$ as η→∞ is relaxed into $F''_0(0)=1$.

III. NUMERICAL METHOD

It is practically tough to get the unique arrangement of when a framework contains fairly non-direct differential conditions. In this manner, we make use of unique methodologies explicitly numerical and investigative methodologies. In this investigation, a numerical methodology has been considered. All the greater especially, the three-phase Lobatto IIIa recipe has been utilized to unravel the association of linearized eigenvalue issues (8-9) with restriction condition (10). It is developed inability of solver in precise BVP4C with the help of code of constrained contrast alongside C^1 collocation polynomial. BVP4C work used to be presented by way of Kierzenka and Shampine (2001) first time to be aware of the two-point limit esteem issues. This collocation is a piece-wise cubic polynomial, and the equation presents a C^1 regular association in that work mistake dedication and control have been developed upon the remaining of the everyday solution.

The relative blunder resistance has been kept [(10)]^(- 5) for this investigation. The suitable assurance of work has been created and saved in sol.x field. BVP4C returns the arrangement as construction and as recognized as sol.y. All matters considered, the arrangement can be gotten from the sol.y comparing subject to sol.x. Additionally, the working gadget of this method has been depicted in chart 1.

IV. RESULT AND DISCUSSION

Before starting of conversation, we seem to be at the numerical consequences of three-phase Lobatto III a recipe with Lund et al., and found in excellent understanding, allude Table 1, which offers us certainty on our technique. It is worth to function that the prerequisites of shaky development of Lund et al. (2019) have been unraveled to evaluation the penalties of double solutions for play out the protection examination. It merits realizing that indications of the littlest eigenvalues show the conduct of bothers of minute around the base circumstance of restrained adequacy. On the off threat that the annoying circulation comes returned to the base kingdom, which implies the indication of the littlest eigenvalues is sure, the stream gets steady. Then again, if the little adequacy annoyances separate from the base state, which implies the indication of the littlest eigenvalues is negative, the circulate is seen as flimsy. It very correctly may also be analyzed from figures 1-2; there exist two scopes of preparations to be specific several preparations and no arrangement. These reaches remember on the estimations of the suction parameter. It tends to be moreover found that the primary arrangement locale partners with the high-quality estimations of the littlest eigenvalues, and in this manner, it is regarded as a constant arrangement. Then again, poor estimations of the littlest eigenvalues can be observed in the district of the subsequent arrangement; from this time ahead, the subsequent arrangement gets steady. It is well worth to make reference to that lone a consistent association has bodily feasibility and can be considered tentatively.

V. CONCLUSION REMARKS:

Remaining Part of stability analysis of dual solution has been discussed, which was not performed by Lund et al., (2019). It is concluded that the first solution is more stable and physically realizable as compare to other solutions from the results of stability analysis.

Table 1:Comparison of skin friction results of Lund et al., (2019) with present results.

		Lund (2019)	Present	Lund (2019)	Present
M	S	1st Solution	1st Solution	2nd Solution	2nd Solution
0.5	3	2.8203848	2.8203848	0.1771243	0.1771243
	2	1.7063214	1.7063214	0.2928932	0.2928932
0	3	2.6165735	2.6165735	0.3819660	0.3819660
	2	1.0019038	1.0019038	1	1

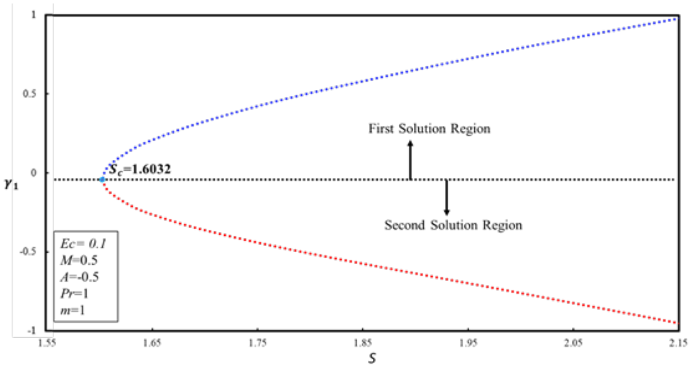

Fig 1: Variation ofγ_1for several values of S

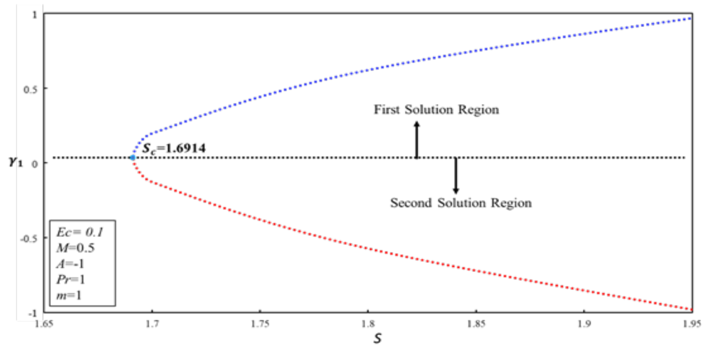

Fig 2: Variation ofγ_1for several values of S.

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